

Hypothetical Learning Trajectory Based on Realistic Mathematics Education in SPLDV Material: Implementation to Accommodate Differences in Junior High School Students' Initial Abilities

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Abstract

This study stems from the challenge of understanding the concept of SPLDV, which is generally still procedural and lacks meaning, as well as the diversity of students' initial abilities that influence their thinking processes and problem-solving strategies. The research method used was qualitative descriptive within the framework of Didactical Design Research (DDR) at SMP Muhammadiyah 5 Surakarta with three ninth-grade students representing high, medium, and low initial abilities as subjects. Data were collected through SPLDV concept comprehension tests and in-depth interviews. The results showed that the application of RME-based HLT helped students understand concepts gradually, starting from real contexts, situational models, mathematical models, to formal representations. Students with high initial abilities were able to fulfill all of Polya's problem-solving indicators, including the evaluation stage. Students with medium initial abilities were able to understand the problem and develop a plan but were not consistent in evaluating the results. Meanwhile, students with low initial abilities were only able to identify basic information without being able to model or implement a solution strategy. Overall, RME-based HLT proved to be effective in facilitating differences in students' initial abilities and improving their understanding of SPLDV concepts through a structured and contextual learning path.

Keywords: Hypothetical Learning Trajectory, Realistic Mathematics Education, SPLDV, initial ability, conceptual understanding.

Introduction

System of Linear Equations with Two Variables (SPLDV) constitute a central topic in algebra because they support students' progression toward more advanced mathematical ideas and contextual problem modeling (Maspupah & Purnama, 2020; Sari & Lestari, 2020). SPLDV trains students to formulate real-world situations into systematic mathematical models so that their reasoning, problem-solving, and understanding of algebraic concepts can develop (Pebrianti & Puspitasari, 2023; Utami & Kusumah, 2023). Students still experience difficulties in converting story problems into mathematical models, determining variables and coefficients, and performing elimination and substitution procedures correctly (Umam & Zulkarnaen, 2022). This condition indicates the need for a more structured and contextual SPLDV learning design so that students' conceptual understanding develops gradually and meaningfully.

Mathematical conceptual understanding may remain weak when instruction emphasizes procedures without connecting symbols to students' contextual meaning-making (Chirove & Ogbonnaya, 2021). RME addresses this by starting from meaningful contexts and guiding students

to build their own models through interaction toward formal mathematics (A'la et al., 2025; Ali, 2022). Recent studies also show that RME-oriented instruction can improve students' understanding and learning outcomes (Tong et al., 2022; Wansurni et al., 2022). In SPLDV, many RME applications still focus on effectiveness, while fewer studies develop a structured HLT that anticipates strategies and obstacles across different initial ability levels (Fahmi et al., 2022; Putriani et al., 2023). Therefore, this study proposes and implements an RME-based HLT to guide students' transitions from contextual situations to formal SPLDV representations while accommodating high, moderate, and low initial ability profiles in one classroom.

Hypothetical Learning Trajectory (HLT) is understood as a hypothetical learning path designed by researchers or teachers, which contains learning objectives, a sequence of tasks or activities, and assumptions about students' strategies and responses to specific learning situations (Nuraida & Amam, 2019). This trajectory helps teachers predict students' thinking patterns, obstacles, and possible misconceptions, so that didactic interventions can be designed more appropriately (Astuti & Wijaya, 2021). The development of HLT in Indonesia on various mathematics topics organizes a series of activities in a gradual and continuous manner so that the transition from concrete contexts to informal models and finally to formal representations can be facilitated effectively (Benu et al., 2024). HLT provides a conceptual framework that helps teachers guide SPLDV learning from contextual situations to formal equations while still allowing room to accommodate differences in students' initial abilities in the classroom.

The differences in students' initial abilities in SPLDV material are evident in the variation in their mastery of prerequisite concepts, such as number operations, algebraic forms, linear equations with one variable, and the ability to present situations in correct symbolic representations (Diaz & Marlina, 2024; Umam & Zulkarnaen, 2022). Classroom conditions generally consist of students with high, medium, and low categories of conceptual understanding and problem-solving abilities, so that SPLDV learning achievements are spread across a relatively wide range of scores (Oktoviani et al., 2019; Palinussa & Tamalene, 2021). Metacognitive research related to SPLDV that groups students based on their initial abilities shows that students with high, medium, and low initial abilities undergo different thinking processes when solving SPLDV problems (Haryaty & Anggo, 2019). The heterogeneous initial ability conditions require that the design of SPLDV learning, including the development of RME and HLT-based learning pathways, explicitly accommodate differences in student readiness so that each ability group can develop optimally.

The Realistic Mathematics Education (RME) approach to SPLDV has been explored mainly through modules and learning media that use realistic contexts, HOTS tasks, and technology, with a focus on tool quality and overall learning outcomes (Daulay et al., 2025; Sektiwulan et al., 2024; Susanti et al., 2020). However, these studies rarely document, step by step, how students move from context to formal SPLDV representations. Evidence on emerging strategies, representational shifts,

and moment-to-moment scaffolding remains limited. International work likewise shows that connecting representations often requires explicit scaffolds to meet high explanatory demands (Zentgraf & Prediger, 2024). Thus, more process-focused research is still needed beyond product quality and score gains.

Previous research has also examined RME implementation in SPLDV for conceptual understanding, problem solving, and learning outcomes, yet systematic Hypothetical Learning Trajectory (HLT) design and analysis remains limited. Studies linking HLT and SPLDV often treat the trajectory as a conceptual product or an analytic lens in specific settings, including RME-linked work (Herawati & Sopiany, 2023; Hidayati et al., 2022). As a result, few studies specify predicted pathways, likely obstacles, and contingency instructional moves. International syntheses emphasize that HLT-based claims require tight alignment between the hypothesized trajectory, tasks, and process evidence (Baroody et al., 2022). Therefore, designing and implementing an RME-based HLT for SPLDV that accommodates mixed initial ability levels remains a strong research opportunity.

This study aims to develop and test a hypothetical learning pathway based on Realistic Mathematics Education on the subject of Two-Variable Linear Equation Systems, which is consciously designed to accommodate differences in the initial abilities of junior high school students. Specifically, this study aims to develop learning activity stages that start from real-world contexts and progress toward formal representations of SPLDV for groups of students with high, medium, and low initial abilities, and to describe how these pathways are implemented in the classroom.

Method

This study employed the Didactical Design Research (DDR) approach, which is a design-based methodology focusing on the iterative development, implementation, and analysis of learning designs. Although qualitative in nature, the study goes beyond a simple descriptive design by systematically aligning with the DDR framework to examine how a Realistic Mathematics Education (RME)-based Hypothetical Learning Trajectory (HLT) supports students' understanding of SPLDV concepts.


The research was conducted at SMP Muhammadiyah 5 Surakarta, involving three ninth-grade students representing different levels of initial mathematical ability (high, medium, and low) as the research subjects. The selection of subjects was based on the results of a preliminary diagnostic test on prerequisite algebraic concepts and recommendations from the classroom teacher, ensuring that the sample represented a diverse range of initial abilities.

In this study, the teacher acted as the primary instructor implementing the RME-based learning design in the classroom, while the researcher played the role of observer and

data collector, responsible for documenting learning interactions, student responses, and instructional adjustments during the process.

Consistent with the DDR paradigm (Suryadi, 2010), the research was carried out in four major stages, namely (1) preliminary analysis, involving the identification of learning challenges in SPLDV through classroom observations and interviews with teachers, (2) didactical design, which developed an RME-based HLT comprising four progressive stages—emergent, model of, model for, and formal, (3) learning Experiment, focusing on classroom implementation of the HLT to observe students' actual learning processes and responses, (4) retrospective Analysis, which entailed qualitative data analysis through data reduction, display, and conclusion drawing to evaluate the effectiveness and alignment of the design with learning outcomes. These stages were designed to provide a structured and replicable process for understanding how the RME-based HLT accommodates variations in students' initial abilities and facilitates their conceptual development in SPLDV learning.

The teacher began the lesson with a context that was familiar to the students by giving them problems that they encounter in their daily lives.



"Children, imagine we go to a food stall, where there is a poster containing two special fried rice menu packages: Economy package: 1 fried rice and 2 iced teas for Rp18,000. Full package: 3 fried rice and 1 iced tea for Rp39,000. How can we know the price of one fried rice and one iced tea?"

Figure 1. realistic questions

The stages of Didactical Design Research (DDR) are presented in Table 1. (Suryadi, 2010).

Table 1. Stages of didactical design research

Stage	Explanation
Preliminary Analysis	This stage involves analyzing the situation to identify problems in learning two-variable linear systems through initial observations and interviews with teachers.
Didactic Design	The four stages of HLT are emergent, model-for, model of, and formal.
Learning Experiment	Implementation of HLT and learning activities
Retrospective Analysis	Qualitative data analysis using reduction, presentation, and conclusion drawing methods.

The collected data were analyzed using content analysis techniques to evaluate the consistency between the designed HLT and its classroom implementation. The units of analysis in this study included (1) students' written work on SPLDV problems, (2) classroom observation notes, and (3) transcripts of student–teacher interactions during learning activities. These data sources were triangulated to ensure the validity of the findings. The primary data sources were students' problem-solving worksheets and recorded discussions, while interview transcripts served as supporting qualitative data to provide insight into students' reasoning and misconceptions.

The analysis followed the stages of data reduction, data display, and conclusion drawing, but was guided by specific indicators derived from both Polya's problem-solving framework and the RME–HLT learning trajectory. These indicators included: (1) students' ability to understand contextual problems, (2) formulation of mathematical models, (3) implementation of problem-solving strategies, and (4) reflection or evaluation of results. Each student's learning process was analyzed to determine how well the actual learning sequence aligned with the expected stages of the RME-based HLT design.

Results and Discussion

The results of the implementation of didactic design research with the following stages:
Stage 1: Preliminary Analysis

Preliminary Analysis in the didactical design research stage is in accordance with the table above, namely in a study conducted on the use of Hypothetical Learning Trajectory (HLT), which is based on Realistic Mathematics Education (RME), in the subject of

Mathematics on the material of two-variable linear equation systems in class VIII of SMP Muhammadiyah 5 Surakarta. Interviews and concept comprehension tests were used to collect data. Students were actively involved in HLT-based learning.

The preliminary analysis stage identified key learning obstacles in understanding systems of linear equations in two variables (SPLDV) through classroom observations, interviews with the mathematics teacher, and diagnostic tests. The analysis revealed that many students still relied on procedural memorization of elimination and substitution methods without connecting them to contextual understanding. Based on these findings, three representative students categorized as high, medium, and low initial ability were selected as research subjects. This categorization provided a basis for designing differentiated learning trajectories within the RME framework.

Stage 2: Didactic Design

Didactic design in Didactical Design Research (DDR) plays a dual role in learning development. Emergent design emerges iteratively from real interactions between students, teachers, and material, thus developing based on the needs and responses of learners. As a "model for," this design becomes a practical guide for teachers to implement effective learning strategies. Meanwhile, as a "model of," the design represents the learning process conceptually, explaining how students construct knowledge and how didactic principles work. Finally, the design can also be formalized into systematic guidelines or theories, so that it can be generalized and used in similar learning contexts.

Within the Didactical Design Research (DDR) paradigm, the didactic design functioned both as a model of and a model for learning. The design was structured according to four levels of the RME-based Hypothetical Learning Trajectory (HLT), (1) situation (contextual level) students engage with realistic problems such as price comparisons at a food stall, (2) model of situation students translate the context into informal representations using symbols or tables, (3) model for knowledge – students begin comparing and generalizing relationships (using elimination or substitution strategies), (4) formal mathematics students represent their reasoning in standard algebraic notation and solve SPLDV systematically. Each stage included predicted student responses and corresponding teacher anticipations, which served as didactic interventions to guide learning progression.

Table 2. Hypothesis of Didactic Design Learning Activities

RME Level	Activity	Prediction	Teacher Anticipation
Situation (attached questions)	<p>The teacher presents a real-life problem at a food stall:</p> <p>Economy Package: 1 fried rice + 2 iced teas = Rp18,000</p> <p>Mukbang Package: 3 fried rice + 1 iced tea = Rp39,000.</p> <p>The teacher asks students to read and write down what they know and what they want to know, then discuss it in small groups.</p>	<p>The students read and understood the context of the story.</p> <p>Some students spontaneously guessed the price of fried rice and iced tea without using a mathematical model.</p> <p>Other students may be confused because the two total prices are different with different combinations of items.</p> <p>Some students began to write down informal relationships such as: "If 3 fried rice dishes plus 1 iced tea cost Rp39,000, then 1 fried rice dish costs approximately Rp10,000?"</p>	<p>The teacher emphasizes the purpose of thinking, not guessing prices, but finding relationships between packages.</p> <p>The teacher asks: "If two packages are different but have some of the same contents, how would you compare them?"</p>
Model of Situation	<p>The teacher guides the students to symbolize the prices of fried rice and iced tea as variables (x and y).</p> <p>The teacher writes two relationships on the board:</p> <p>1) $x + 2y = 18,000$ (Economy Package)</p> <p>2) $3x + y = 39,000$ (Mukbang Package)</p>	<p>Some students immediately accept the symbolic representation.</p> <p>Others may be confused about "why use letters?" and feel more comfortable with the words "price of fried rice" and "price of iced tea."</p>	<p>The teacher emphasizes that letters do not replace numbers, but represent unknown prices.</p> <p>The teacher can ask: "If 'x' is the price of fried rice, then how much would two plates of fried rice cost?"</p>

	<p>The teacher ensures that the students understand the meaning of the symbols.</p>	<p>Students may rewrite it in their own style, for example, "$ng + 2t = 18,000$" ($ng =$ fried rice, $t =$ tea).</p> <p>Informal models (self-developed models) emerge in the form of simple sentences or tables.</p>	
Model for Knowledge	<p>The teacher guides students to find a way to compare two packages so that one item "disappears."</p> <p>Teacher: "If we want to know the price of iced tea, how about we make the amount of fried rice in the two packages the same first?"</p> <p>The teacher gives students the opportunity to double one of the packages and compare the results.</p>	<p>Some students took the initiative to double the Economy Package to $3x$ so that the fried rice portions were the same (3 vs. 3).</p> <p>Students write a comparison table:</p> <ul style="list-style-type: none"> • Savings $\times 3 \rightarrow 3$ fried rice + 6 iced teas = Rp54,000 • Mukbang $\rightarrow 3$ fried rice + 1 iced tea = Rp39,000 <p>The students compare and find: "If the 3 fried rice dishes are the same, then there is a difference of Rp15,000 for the 5 iced teas. So, 1 iced tea costs Rp3,000."</p> <p>After that, they return to the original equation and find that fried rice costs Rp12,000.</p>	<p>The teacher emphasizes the comparison step using contextual language: "When two packages have the same amount of fried rice, we can see that the price difference comes from the drinks."</p> <p>The teacher asks progressive questions: – "What is the same about these two packages?" – "What is different about them?" – "If the price difference is Rp15,000 for 5 glasses of iced tea, how much does 1 glass cost?"</p>

		Some students who did not understand tried trial and error.	
Formal Mathematics	<p>The teacher rewrites the student's steps into formal SPLDV form:</p> <p>1) $x + 2y = 18,000$ (1) 2) $3x + y = 39,000$ (2)</p> <p>The teacher asks: "How can we make the amount of fried rice in the two packages the same?" (\rightarrow multiply (1) $\times 3$).</p> <p>The teacher continues with the formal steps: $3x + 6y = 54,000$</p> <p>Subtract (2): $(3x + 6y) - (3x + y) = 54,000 - 39,000$ $\rightarrow 5y = 15,000 \rightarrow y = 3,000 \rightarrow x = 12,000$.</p>	<p>Students begin to realize that the steps of "multiplying and subtracting" can be formulated in a general symbolic form.</p> <p>Some students call it "eliminating variables," others call it "equalizing first so that it can be subtracted."</p> <p>Some students may make a mistake with the sign when subtracting (e.g., $6y - y = 7y$).</p>	<p>The teacher emphasizes the concept of signs: use different colors for the variables that are eliminated.</p> <p>The teacher asks again about the meaning of the result: "If $x = 12,000$ and $y = 3,000$, what does that mean in context?"</p>

Stage 3: Experiment


The learning experiment stage is the stage where the didactic design created in the prospective analysis stage is tested directly in the classroom. The aim is to see how students actually learn, how the learning situation develops, and whether the design can facilitate the expected learning trajectory. At this stage, researchers not only observe the success or failure of the design, but also trace student responses, identify learning obstacles that arise, and note the necessary teacher interventions. This experiment is flexible because the learning process of students often develops unexpectedly. Therefore, this stage is closely related to the concept of emergent modeling, which explains how the model used by students develops from a model of (concrete/initial representation) to a model for (formal or general thinking tool).

During the learning experiment, the RME-based HLT was implemented in class and analyzed through content analysis involving three primary units of analysis, (1) students written problem-solving work, (2) classroom observation records, and (3) transcripts of student–teacher dialogue. The indicators of analysis adapted from Polya’s problem-solving stages and HLT phases—including: understanding the problem (contextualization), formulating mathematical models, executing solution strategies, and evaluating results. The findings show a progressive alignment between the designed HLT and actual student responses.

Students with high initial ability followed the entire learning trajectory smoothly from recognizing contextual relationships to formalizing algebraic models demonstrating a full correspondence between predicted and observed learning pathways. Students with medium initial ability successfully reached the model for knowledge stage but exhibited inconsistencies in verification, indicating partial alignment between the predicted and actual trajectory. Students with low initial ability remained largely at the situation and model of situation levels, requiring frequent teacher scaffolding. Their responses confirmed the need for anticipatory guidance embedded in the HLT design.

This connection between students’ work and the trajectory illustrates that the didactical design not only structured learning activities but also shaped students’ cognitive progression. Each student’s movement across HLT levels reflected how RME contexts facilitated conceptual reinvention from informal reasoning to formal algebraic representation.

Table 3. Learning Activities Learning Experiments

Activity Name (RME Level)	Teacher Activity (Steps and Scaffolding)
1. Situation	<p>The teacher begins the lesson with a context familiar to the students: "Children, imagine we are at your favorite restaurant. There are two special fried rice menu packages: Economy Package: 1 fried rice and 2 iced teas for Rp18,000. Full Package: 3 fried rice and 1 iced tea for Rp39,000. How can we find out the price of one fried rice and one iced tea?"</p>  <p>The teacher asks the students to reread the text, write down the 'known' and 'unknown' information, and discuss it in small groups (3–4 students).</p> <p>The teacher asks openly:</p> <ul style="list-style-type: none"> – “What do you know about this story?” – “Is there a connection between these two packages?” – “What if we try to compare them?” <p>The teacher ensures that all students understand the situation by repeating the questions and providing explanations if anyone is confused.</p>
2. Model of Situation	<p>The teacher guides the students to a mathematical model with guided questions: "If we use letters to represent prices, how would we write down the relationship?"</p> <p>The teacher writes the results of the discussion on the board:</p> <ol style="list-style-type: none"> 1) $x + 2y = 18,000 \rightarrow$ Economy Package 2) $3x + y = 39,000 \rightarrow$ Full Package <p>The teacher ensures understanding of the symbols: "x represents the price of fried rice, y represents the price of iced tea."</p>

<p>3. Model for Knowledge</p>	<p>The teacher does not directly teach the elimination formula, but guides students to discover the idea themselves through exploration:</p> <p>Teacher: "If we want to know the price of one type of item, can we make the amount of fried rice the same in both packages?"</p> <p>Teacher prompts the idea: "If the Economy Package only has 1 fried rice, how can we make it 3 like the Full Package?"</p> <p>Students are asked to multiply the Economy Package by 3 and rewrite the comparison.</p>
<p>4. Formal Knowledge</p>	<p>The teacher rewrites the students' steps into a formal mathematical form:</p> $1) x + 2y = 18,000 \quad (1)$ $2) 3x + y = 39,000 \quad (2)$ <p>Multiply (1) $\times 3$ so that the coefficient of x is the same:</p> $3x + 6y = 54,000 \quad (3)$ <p>Then subtract (3) – (2):</p> $(3x + 6y) - (3x + y) = 54,000 - 39,000$ $\rightarrow 5y = 15,000$ $\rightarrow y = 3,000$ <p>Substitute into (1):</p> $x + 2(3,000) = 18,000 \rightarrow x = 12,000.$ <p>The teacher emphasizes the meaning of elimination: "We make one component disappear by equating the amounts." The teacher gives similar exercises to reinforce the concept.</p> <p>The teacher guides students toward the general concept of SPLDV through guiding questions:</p> <ol style="list-style-type: none"> 1. "What is a system of two variable equations?" 2. "How do you find the values of x and y so that both equations are true?" 3. "What does this result mean in the context of the cafeteria problem?" <p>The teacher concluded that a System of Two Variable Linear Equations (SPLDV) consists of two interrelated equations and has two letters (usually x and y).</p> <p>The correct values of x and y are pairs of numbers that make both equations true at the same time.</p> <p>The teacher gave another example of a problem involving the multiplication rule to reinforce the students' understanding.</p> <p>Students were asked to solve the problems given by the teacher.</p>

The results of the concept comprehension test show that students achieved an average increase in scores after implementing RME-based HLT. High-ability students had the highest scores compared to students with moderate abilities. Students with moderate abilities scored higher than students with low abilities. This increase in scores indicates that the implementation of RME-based HLT has a positive impact on student comprehension based on Polya's problem-solving indicators. Using Polya's problem-solving approach, which includes (1) Understanding the Problem, (2) Devising a Plan (Formulating a Plan/Strategy), (3) Carrying Out the Plan (Implementing the Plan), and (4) Looking Back (Reviewing & Evaluating), the following analysis results were obtained.

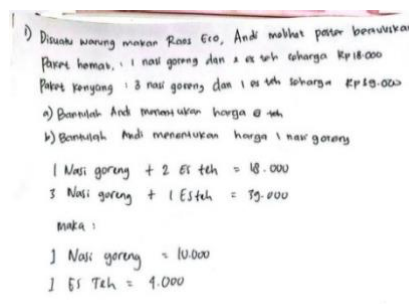


Figure 2. Results of Students' Work with Low Initial Ability

The figure shows the work results of students with low initial abilities. From this display, it can be seen that the subjects were only able to rewrite the problem according to the story without converting it into a mathematical model. This condition shows that the subject has not met Polya's problem understanding stage indicator. This is reinforced by the interview results, where when the researcher asked about mathematical modeling, the subject replied that he did not know and could not model the problem. In addition, the subject also did not solve the problem correctly; after writing down the question, the subject immediately wrote down the answer without going through the process of plan development or plan implementation. Thus, the subject did not meet the indicators of planning and implementing a plan in the problem-solving stage according to Polya's. Based on the results of the analysis conducted with students with low initial ability levels, it can be concluded that students with low initial ability levels have not been able to meet the problem-solving indicators (Jannah et al., 2025)

D) Disuatu warung makan Rasi Eco, Andi melihat poster bertuliskan
 Paket hemat : 1 nasi goreng dan 2 es teh seharga Rp18.000.
 Paket kenyang : 3 nasi goreng dan 1 es teh seharga Rp39.000.

a) Bantulah Andi menentukan harga es teh.
 b) Bantulah Andi menentukan harga 1 nasi goreng.

Jawab:

① a)
$$\begin{array}{r} 1x + 2y = 18.000 \quad | \times 3 \\ 3x + 1y = 39.000 \quad | \times 1 \\ \hline = 3x + 6y = 54.000 \\ 3x + 1y = 39.000 \quad - \\ \hline 0 + 5y = 15.000 \\ y = \frac{15.000}{5} \\ y = 3.000 \end{array}$$

b)
$$\begin{array}{r} 3x + 1y = 39.000 \\ 3x + 3.000 = 39.000 \\ 3x = 39.000 - 3.000 \\ 3x = 36.000 \\ x = \frac{36.000}{3} \\ x = 12.000 \end{array}$$

Figure 3. Results of Students' Work with Moderate Initial Ability

The figure shows the work results of subjects with moderate initial ability. These subjects were able to perform mathematical modeling in accordance with Polya's indicators at the problem understanding stage. The subjects were also able to determine the answer using the selected methods, namely elimination and substitution, thus meeting the indicators for the planning and implementation stages in problem solving according to Polya. However, the subject did not carry out the last indicator, namely evaluating and reflecting, as evidenced by the interview results that the subject was unable to recheck the answers obtained.

D) Disuatu warung makan Rasi Eco, Andi melihat poster bertuliskan
 Paket hemat : 1 nasi goreng dan 2 es teh seharga Rp18.000.
 Paket kenyang : 3 nasi goreng dan 1 es teh seharga Rp39.000.

a) Bantulah Andi menentukan harga es teh.
 b) Bantulah Andi menentukan harga 1 nasi goreng.

Jawab:

~~$$\begin{array}{r} 1x + 2y = 18.000 \\ 3x + 1y = 39.000 \end{array}$$~~

Paket hemat : $1x + 2y = 18.000$
 Paket kenyang : $3x + 1y = 39.000$

$$\begin{array}{r} 1x + 2y = 18.000 \\ 6x + 2y = 78.000 \\ \hline -5x = -60.000 \\ x = \frac{-60.000}{-5} \\ x = 12.000 \end{array}$$

harga 1 nasi goreng adalah: 12.000

Substitusi harga nasi goreng

$$\begin{array}{r} 1x + 2y = 18.000 \\ 12.000 + 2y = 18.000 \\ 2y = 18.000 - 12.000 \\ 2y = 6.000 \\ y = \frac{6.000}{2} \\ y = 3.000 \end{array}$$

Substitusi harga es teh

$$\begin{array}{r} 12.000 + 2y = 18.000 \\ 2y = 18.000 - 12.000 \\ 2y = 6.000 \\ y = \frac{6.000}{2} \\ y = 3.000 \end{array}$$

②. harga 1 nasi goreng

$$\begin{array}{r} 1x + 2y = 18.000 \\ 3x + 1y = 39.000 \\ y = 3.000 - 3x \\ x + 2(3.000 - 3x) = 18.000 \\ x + 7.000 - 6x = 18.000 \\ -5x = 18.000 - 7.000 \\ -5x = -60.000 \\ x = \frac{-60.000}{-5} \\ x = 12.000 \end{array}$$

Figure 4. Results of Students' Work with High Initial Ability

The image shows the results of the work of subjects with high initial abilities. The subjects were able to carry out each of Polya's problem-solving indicators. The subjects were able to write down mathematical models of the existing problems. The subjects were also able to compile and carry out plans well, namely by using the elimination and substitution methods. The subjects were also able to explain the correct and detailed solutions to the problems that had been written down.

Stage 4: Retrospective Analysis

The results of the work and interviews conducted show that subjects with high initial abilities who have a deep understanding of concepts and high speed tend to have detailed and correct answers. Subjects with moderate initial abilities have a deep understanding but lack detail in their writing because they need a little guidance. Meanwhile, subjects with low initial abilities have a lack of understanding, so they need a lot of guidance and direction in solving the problems given.

The HLT learning process helps students understand concepts gradually. The difference between subjects with high, moderate, and low initial abilities lies in their depth of understanding and speed in completing tasks. In line with this Marselina & Roesdiana (2019) state that differences in students' initial abilities tend to affect their ability to understand problems, plan and implement strategies, and check results. Meanwhile, students with moderate initial abilities can understand problems but are not yet consistent in planning or implementing strategies, and their mathematical arguments are still limited. Students with low initial abilities can only identify basic elements, often skip important steps, and have difficulty making arguments or connecting concepts.

The retrospective analysis synthesized results across all data sources to assess how well the implemented instruction aligned with the designed HLT. Triangulation between student work, observations, and interviews confirmed that the RME-based HLT effectively supported differentiated learning paths corresponding to students' initial abilities. The iterative nature of DDR allowed the researcher to refine instructional strategies, especially in bridging the model for and formal stages for students with moderate abilities. Overall, the analysis validates that the designed HLT operated as both a predictive and explanatory tool, offering insight into how real-world contexts and guided modeling foster conceptual understanding of SPLDV.

Conclusion and Suggestion

The results of the implementation show that the use of RME-based HLT is effective in overcoming differences in students' initial abilities in understanding the concept of two-variable linear equation systems. The research shows that the HLT stages improve students' understanding of SPLDV material with high, moderate, and low abilities. Although students' comprehension abilities differ, the RME-based HLT method is able to help students solve problems.

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