Designing Mathematics Instruction Based on Problem-Based Learning to Improve the Mathematical Communication Skills of Eighth-Grade Junior High School Students

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Abstract

This study aims to analyze the need to develop a Problem-Based Learning (PBL)-based mathematics instructional design to improve the mathematical communication skills of eighth-grade junior high school students. The study employed a descriptive approach and involved 31 eighth-grade students and a mathematics teacher at junior high school in Jakarta. Data were collected through observation, a mathematical communication skills test, a questionnaire, and interviews. The results of the study indicate that students' mathematical communication skills remain low, particularly in constructing mathematical models, representing ideas in graphs, and explaining solutions in their own words. The average test score of 20.11 indicates that most students have not yet met the expected proficiency indicators. Furthermore, instruction remains teacher-centered, resulting in students being less active in communicating mathematical ideas. The results of the curriculum and conceptual analysis identified "Equations of Straight Lines" as a suitable topic to be developed through PBL-based instructional design. Thus, the results of this needs analysis serve as the basis for designing an instructional approach capable of facilitating the improvement of students' mathematical communication skills.

Keywords: Instructional Design, Mathematical Communication Skills, Problem-Based Learning, Equations of Straight Lines

Introduction

The quality of a country's human resources is determined by the quality of education available, which plays a crucial role in developing human resources with 21st-century skills. According to "Partners for 21st Century Skills," the skills required in this era include a) critical thinking and problem-solving, b) communication and collaboration, and c) creativity and innovation. Individuals must possess these skills to compete on a global scale, so the education sector must be able to provide learning experiences that develop 21st-century skills (Voogt & Roblin, 2010). Education in this century aims to prepare students to succeed in a rapidly changing world, where adaptability, creativity, and communication skills are becoming increasingly important (Muna dkk., 2024). This transformation calls for a more meaningful paradigm shift and a keen analysis of learning, and one of the key areas that must be improved in this century is the development of mathematics education (Susilowati et al., 2020)

The importance of mastering mathematics is evident in Article 37 of Law No. 20 of 2003 on the National Education System, which stipulates that mathematics is a required subject for students at the elementary and secondary levels. Mathematics instruction in schools is expected to develop students' problem-solving abilities and skills to a higher level and to serve as a means for students to practice their mathematical abilities. As supported by Ministry of Education Regulation No. 22 of 2006, the goal of mathematics instruction is for students to be able to communicate their ideas through symbols, tables, diagrams, or other media to clarify existing situations or problems. This is in line with one of the skills students must possess, namely mathematical communication skills.

Supartik and Pasaribu (2021) explain that communication serves as a tool for conveying messages with specific purposes. According to Ariani (2018), mathematical communication skills refer to the ability to convey mathematical ideas or concepts both orally and in writing, as well as the ability to understand and accept others' ideas in order to deepen one's understanding of the material. This is in line with the view of Ahmad and Nasution (2018), who state that mathematical communication involves writing and expressing mathematical concepts using symbols or mathematical language. Mathematical communication skills enable students to solve math problems in various ways more easily (Purnamasari & Afriansyah, 2021). Thus, it can be concluded that mathematical communication skills are the ability to convey ideas in the language of mathematics. When students learn mathematics, they are expected to be able to communicate their understanding.

According to Reni and Gordah (2013), indicators of mathematical communication skills are grouped into three categories: written text, drawing, and mathematical expression. Meanwhile, the indicators of mathematical communication skills presented by I. P. Sari (2017) are grouped into five categories, namely expressing mathematical ideas or problems through writing, representing mathematical ideas or problems visually in the form of graphs, diagrams, or tables; using mathematical terms, notations, and symbols to present mathematical ideas; interpreting mathematical ideas or problems in their own words; and drawing conclusions from mathematical statements. Mathematical communication skills are so important that, without good communication, students will find it increasingly difficult to understand the material, which will have a negative impact on their learning outcomes (Nursamsih Lubis & Rahayu, 2023).

However, empirical data show that students' mathematical communication skills currently remain in a concerning state. Fauziah et al. (2022) revealed in their study that the mathematics scores achieved by students in Indonesia are still significantly below the global average, with the majority of students struggling when faced with non-routine assessment tasks that require higher-order thinking. These deficiencies in mathematical communication skills prevent students from achieving the expected learning outcomes of the teaching-learning process.

The main obstacle students often face arises when they are presented with math problems that are different from or use visualizations that differ from the examples explained when the material was covered in class. Students tend to think convergently—that is, they simply imitate and mechanically copy the problem-solving steps provided by the teacher without understanding the essence of the concept (Hidayat, 2012). As a result, when assessments or exams present new problems that differ from the practice problems, students become disoriented, unsure of where to begin solving the problem, and ultimately choose to give up. This pattern occurs because students are not yet accustomed to conceptualizing mathematical ideas into verbal or written representations on their own, so their mathematical communication skills in problem-solving are still at a low level and need to be retrained immediately (Ulandari et al., 2019).

This is consistent with the issues observed at junior high school in Jakarta. Research has shown that students' mathematical communication skills remain very low. This is because students have not yet mastered mathematical modeling, are not yet proficient at drawing graphs, and are not yet able to provide answers to problems using their own words, as they struggle to connect mathematical ideas to problem-solving. This is supported by the results of interviews with teachers, who stated that students are not yet fully capable of solving problems related to graphs. They still have difficulty drawing and interpreting them.

One of the math topics that students find challenging when applying mathematical concepts to the given problems is Linear Functions, or Straight-Line Equations (SLE). A total of 53.8% identified SLE material as difficult to understand. This material is considered difficult to understand because students are still unable to grasp the concepts and formulas used, and they struggle to model problems particularly when drawing graphs. This is also one of the consequences of poor mathematical communication skills. Furthermore, interviews with teachers revealed that instruction on PGL material tends to remain teacher-centered and is delivered directly by introducing mathematical notation, formulas, and graphs before students try them on their own. Based on this, Sari (2017) states that learning using this method becomes meaningless because students do not see how the material directly applies to problems in their daily lives.

To break this cycle of cognitive barriers, the achievement of ideal mathematics learning objectives depends heavily on how classroom learning scenarios are designed. The goal of improving mathematical communication skills will only be effectively achieved if instructional planning and models are specifically designed to encourage active participation, discussion, and critical thinking among students (Rahmawati, 2018). One of the most relevant alternative strategies for the characteristics of the 8th-grade junior high school curriculum is the integration of the Problem-Based Learning (PBL) model.

Research conducted by Ningrum (2017) indicates that the PBL model is one of the instructional models that can serve as a means of developing students' mathematical communication skills. This is further supported by research conducted by Sartika (2018), which found that PBL has an impact on students' mathematical communication skills. The PBL model presents students with problems, prompting them to seek solutions to the challenges they face. The key characteristics of PBL which is student-centered and based on real-life problem-solving compel students to step out of their passive zone, so that they no longer simply accept formulas at face value from teachers, leading students to gradually realize their own thinking potential (Azizah & Granita, 2020).

According to Arends (2012), the PBL model is a learning model that uses authentic problems as the starting point for learning, this encourages students to investigate, seek information, discuss, and formulate solutions to the problems they face. Through the PBL framework—which includes problem orientation, student organization, individual and group investigation, presentation of work, and evaluation of the problem-solving process—students have the opportunity to construct their own knowledge.

Active learning models such as PBL are essential for addressing challenges in improving students' mathematical communication skills (Purnomo & Wahyudi, 2021). Consequently, PBL-based mathematics instruction centered on real-world contextual problems has been shown to encourage students to interact with one another, negotiate ideas, document mathematical models, and present their logical arguments in front of the class (Simamora et al., 2017). While prior studies have broadly demonstrated the benefits of PBL, this study offers novelty by specifically examining a structured and adaptive PBL instructional design tailored to real-world linear equation problems, providing a targeted framework to address specific communication barriers in algebraic reasoning among eighth-grade students. Through the implementation of this approach, it is hoped that barriers to mathematical communication can be overcome and students' mathematical abilities can improve significantly.

Method

This study uses a descriptive approach to identify the need for a Problem-Based Learning (PBL)-based mathematics instructional design aimed at improving the mathematical communication skills of eighth-grade students in junior high school. This study focuses on the needs analysis stage by collecting information regarding students' mathematical communication skills and the conditions of mathematics instruction at the school. The results of this needs analysis serve as the basis for developing a PBL-based instructional design.

This study was conducted at junior high school in Jakarta, with the research subjects consisting of eighth-grade students and a mathematics teacher. Eighth-grade students and a mathematics teacher were selected as research subjects because linear equations are a core component of the eighth-grade curriculum, where fundamental algebraic communication barriers frequently arise. The students served as the source of data regarding their mathematical communication skills. Meanwhile, teachers serve as the primary sources of information regarding the learning process, student characteristics, the learning strategies used, and the challenges faced in developing students' mathematical communication skills.

Research data were collected using two methods: tests and interviews. Tests were administered to students to obtain an initial assessment of their mathematical communication skills. The test instrument was designed in the form of essay questions based on indicators of mathematical communication skills (such as expressing mathematical situations or ideas in the form of mathematical models or symbols, explaining mathematical concepts in writing, and interpreting and evaluating mathematical ideas).

On the other hand, semi-structured interviews were conducted with mathematics teachers to gather information about the implementation of instruction, the difficulties students face in communicating their mathematical ideas in writing, the instructional models that have been used to date, and the teachers' needs for instructional designs that can facilitate the improvement of students' mathematical communication skills. To ensure the validity of the research instruments, including both the essay test and the interview guidelines, content validation was conducted by three experts in the field of mathematics education. These experts evaluated the construct, content, and language clarity of the instruments using validation sheets, and their feedback was used to revise and refine the instruments before field administration.

The test data were analyzed descriptively by calculating the scores obtained by students on each indicator of mathematical communication skills, which were then described to identify aspects that still require improvement. Interview data were analyzed using qualitative data analysis techniques according to Miles, Huberman, and Saldaña (2014), which include data condensation, data presentation, and drawing conclusions and verification. The results of the analysis of these tests and interviews were then integrated to serve as the basis for designing a PBL-based mathematics curriculum tailored to the students' needs and the learning conditions at that school.

Results and Discussion

Analysis of Mathematics Learning Needs

In the initial stage, a needs analysis was conducted for both students and teachers. The purpose of the needs analysis was to gather and identify information regarding the problems or obstacles faced by teachers and students in mathematics learning, as well as to assess the needs of both groups. Data collection for the needs analysis was conducted through observations of mathematics lessons in the classroom, the distribution of questionnaires to 31 eighth-grade students, interviews with mathematics teachers, and cognitive tests administered to students.

Based on the results of interviews with teachers, students still struggle to connect the information obtained from the problems presented with the concepts needed to solve them, especially when given problems involving graphs. In addition, students' lack of motivation leads to low self-confidence. Most students are passive, quiet, and merely take notes, while only a small number actively participate in the learning process. When it comes to discussions, some students still struggle to communicate effectively both verbally and in writing and are afraid to express their ideas; they are accustomed to simply copying others' work.

One of the math topics that students find difficult to learn is linear equations. In interviews conducted, teachers explained that students are not yet fully capable of solving problems related to graphs. They still have difficulty drawing and interpreting them. This is consistent with the fact that students' limitations in mastering the concepts and recognizing the practical applications of linear equations are directly related to weaknesses in their mathematical communication skills; therefore, to determine the depth of their understanding of these concepts, an instrument is needed to measure students' mathematical communication skills.

This assessment was conducted because mathematical communication skills involve students' ability to represent real-world situations using diagrams, graphs, or algebra, as well as their ability to explain mathematical concepts in a logical sequence. Through appropriate assessment of the material on Linear Equations, Educators can identify where students' misconceptions lie—whether at the stage of representing graphs or when testing the validity of a mathematical argument. According to the results of the mathematical communication skills test on the topic of linear equations that was administered, nearly all students were unable to answer the questions correctly.

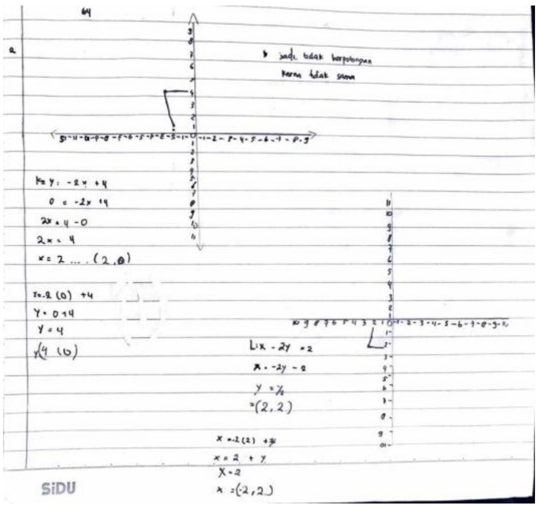

Figure 1. Students' failure to answer one of the questions

Based on these results, it is evident that the students failed to draw a straight line graph. Instead of drawing two complete straight lines extending across the Cartesian plane, they drew two small, separate sketches of the coordinate plane and simply drew lines at right angles**.** From this example, it can be concluded that the students are still unable to solve problems related to drawings, so it can be said that their communication skills in drawing are still low.

The results of the cognitive ability test administered to 31 eighth-grade students showed an average score of 20.11, with the highest score being 50 and the lowest being 3.33. The results show that students have not yet met the indicators of mathematical communication skills regarding the topic of linear equations. Niasih et al. (2019) explain in their study that one of the reasons students are unable to solve problems is that they have not yet mastered how to model mathematical concepts, have not yet mastered graphing effectively, and are not yet able to provide explanations using their own words because students have difficulty connecting mathematical concepts to problem-solving.

Curriculum Analysis as the Basis for Developing the Instructional Design

Next, a task and concept analysis was conducted based on the school's curriculum and materials aligned with the learning outcomes (CP). Following that, the learning objectives—derived from the task and concept analysis—were defined as objectives that describe the skills or competencies students can demonstrate. The textbooks used by teachers and students were also analyzed to identify the content standards to be taught, so that the expected competencies for each topic could be achieved.

Based on the results of the curriculum document analysis, the topic of Linear Functions/Linear Equations (PGL) was selected as the focus for eighth-grade junior high school students in this study. The selection of this topic was based on the characteristics of the PGL material, which is rich in real-life contextual problems, thus it has great potential to stimulate students' mathematical communication skills. In Phase D Learning Outcomes (CP), students are expected not only to understand the FGL concept but also to be skilled at transforming contextual problems into mathematical models (mathematical representations), explaining solution strategies, and interpreting and communicating the results of those solutions back into the original problem context.

Concept Analysis

A conceptual analysis was conducted to identify the essential ideas in the linear functions curriculum, which will serve as the foundation for developing a Problem-Based Learning (PBL)-based instructional design (Gravemeijer et al., 2017). This analysis aims to ensure that the material is presented in a logical sequence, is appropriate for the students' characteristics, and is capable of facilitating the improvement of their mathematical communication skills (NCTM, 2000).

The results of the analysis show that the topic of linear functions consists of several interrelated key concepts. These concepts include the definition of a function and function notation, slope (gradient), the equation of a linear function, graphical representation on the Cartesian plane, and the modeling and interpretation of linear functions in solving everyday problems. Mastery of each of these concepts is an essential prerequisite for students to be able to communicate their mathematical ideas systematically, whether in visual (graphs), symbolic (equations), or verbal (written explanations) form (Triana et al., 2020). Therefore, the material should be presented in stages, progressing from simple conceptual understanding to more complex modeling applications.

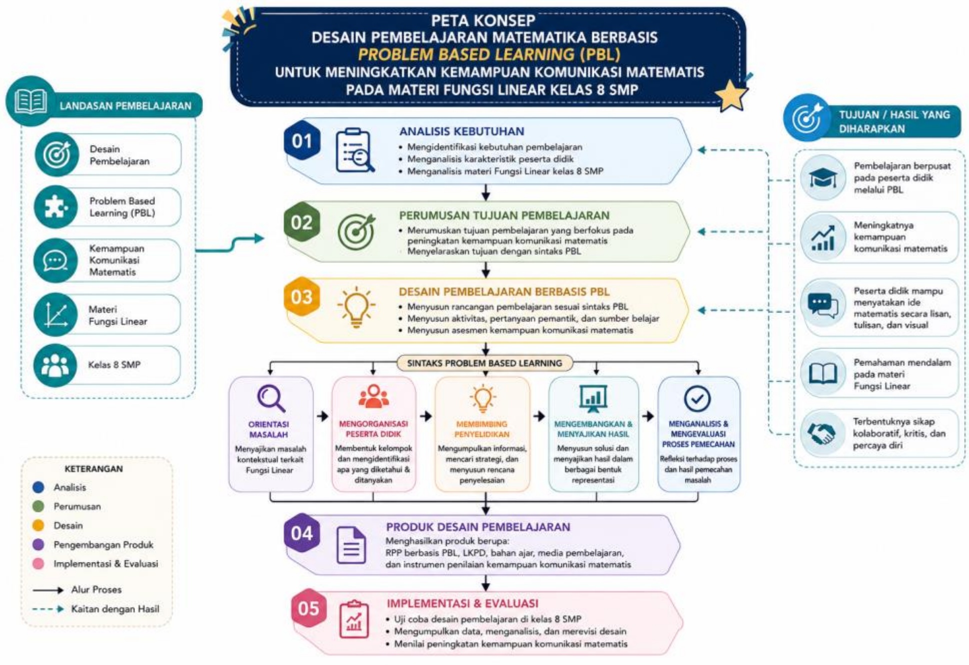

Figure 2. Concept Map

In addition to the main concepts, this analysis also reveals several prerequisite concepts that students must master before studying linear functions, such as algebraic operations, the Cartesian coordinate system, functions in general (relations and functions), and linear equations in one variable (Kieran, 2007). Mastery of these prerequisite concepts is crucial to students' success in transforming real-world situations into mathematical models and present them in graphical form. If these prerequisite concepts are not well understood, students tend to encounter difficulties when determining slopes, plotting coordinate points, and performing algebraic manipulations in the process of mathematical communication.

Based on the results of the pre-test analysis, several misconceptions were identified that students still hold. These challenges include errors in interpreting the meaning of the slope, an inability to transform contextual problems into graphical representations or linear equations, and difficulties in explaining the physical meaning of the x- and y-intercepts on a function graph. These findings indicate that learning should not be limited to calculation procedures but should provide students with opportunities to explore the meaning of each concept through real-world problem-solving (Skemp, 1976; Van de Walle et al., 2019).

The characteristics of linear function material, which are closely related to everyday phenomena, make it ideal for development through the Problem-Based Learning model (Arends, 2012; Barrows, 1996). Contextual problems such as transportation fares, growth rates, changes in water volume over time, or the depreciation of goods can serve as learning stimuli. These stimuli will encourage students to identify information, organize their ideas into tables or graphs, formulate mathematical equations, and communicate their conclusions in writing. Thus, the results of this conceptual analysis serve as the primary reference for determining the sequence of material, designing classroom activities, developing student worksheets (LKPD), and creating assessment tools that support the strengthening of students' mathematical communication skills (Suryadi, 2019).

Conclusion and Suggestion

The results of the needs analysis indicate that the mathematical communication skills of eighth-grade students at junior high school in Jakarta are still relatively low. The difficulties students face include their ability to represent mathematical ideas in the form of graphs, mathematical models, symbols, or written explanations. In addition, students also face challenges in connecting contextual problems with mathematical concepts, interpreting graphs, and presenting reasons or solutions using their own words. This is supported by the results of the mathematical communication skills test, which showed an average score of 20.11, indicating that most students have not yet met the expected indicators of mathematical communication skills. The results of observations and interviews with teachers show that the learning process is still teacher-centered, as a result students tend to be passive, lack the confidence to express their opinions, and are not yet accustomed to discussing or communicating mathematical ideas, either orally or in writing. In the unit on Linear Equations, students also continue to struggle with understanding the concept of slope, drawing and interpreting graphs, and transforming contextual problems into mathematical models. Curriculum analysis and conceptual analysis indicate that the topic of "Equations of Straight Lines" has characteristics that make it suitable for development through a Problem-Based Learning (PBL) instructional design. This topic allows students to engage in solving contextual problems, thereby helping them develop the ability to represent, explain, and communicate mathematical ideas in a more meaningful way. The conceptual analysis also successfully identified prerequisite concepts, the sequence of material, and various misconceptions that need to be addressed in the development of instructional designs. Thus, the results of this study provide a strong foundation for the development of mathematics instructional designs based on Problem-Based Learning as an effort to improve students' mathematical communication skills.

Based on the research findings, the development of Problem-Based Learning-based instructional designs should be continued through the validation, implementation, and evaluation stages to test their validity, practicality, and effectiveness in improving students' mathematical communication skills. Mathematics teachers are advised to implement instruction focused on contextual problem-solving and collaborative discussion, and providing students with opportunities to express their ideas and explain their mathematical thought processes through various forms of representation. In addition, the development of instructional materials, student worksheets (LKPD), and assessment instruments should be aligned with the Problem-Based Learning framework and indicators of mathematical communication skills so that learning becomes more meaningful and active and can improve the quality of students' mathematical communication.

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